$$\sum_{n=0}^{\infty} (x-1)^n = \int 1 + (x-1) + (x-1)^2 + (x-1)^3 + \ldots = \frac{1}{1-(x-1)}$$

$a = 1 \quad r = (x-1)$

$$= \int_0^x \frac{1}{2-x}$$

69. $|r| < 1 \quad |x-1| < 1$

(A) $-1 < x-1 < 1$

$0 < x < 2$

$$-\ln \frac{|x-2|}{2} = -(\ln|x-2| - \ln 2)$$

70. E

71. $-\ln|2-x| + c = -\ln|x-2| + c$

$+ \ln|2|$

$$\ln \frac{2}{|x-2|}$$

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63.

$$\int \frac{1}{x} = \frac{1}{1+(x-1)} = \int 1 - (x-1) + (x-1)^2 - (x-1)^3$$

$a = 1 \quad r = -(x-1)$

$$\ln x = x - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \ldots + c$$

$x = 1$
$y = 0$

$0 = 1 + c$

$-1 = c$

$$(x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} \ldots$$

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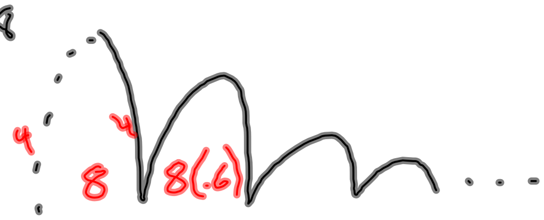

$$\frac{8}{1-.6} - 4$$

$a = 8$

$r = .6$

21. $\sum_{n=0}^{\infty} 2^n x^n = 1 + 2x + 4x^2 + 8x^3 \dots = f(x)$

27. $f'(x) = 2 + 8x + 24x^2 + \dots = \sum_{n=1} 2^n \cdot n x^{n-1}$

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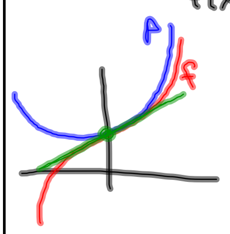
9.2 Taylor Series

given $f(x)$, find $p(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \dots$

$f(x) \simeq p(x)$ for x close to 0



$f(0) = p(0)$

$f(0) = a_0$

$f'(0) = p'(0)$

$f'(0) = a_1$

$f''(0) = p''(0)$

$f''(0) = 2a_2$

$\frac{f''(0)}{2} = a_2$

$p'(x) = a_1 + 2a_2 x + 3a_3 x^2 + \dots$

$p''(x) = 2a_2 + 3 \cdot 2 a_3 x + \dots$

$p'''(x) = 3 \cdot 2 a_3 + \dots$

$p'''(0) = 3 \cdot 2 a_3$

$f'''(0) = p'''(0) = 3 \cdot 2 a_3$

$\frac{f'''(0)}{3 \cdot 2 \cdot 1} = a_3$

$a_4 = \frac{f^4(0)}{4 \cdot 3 \cdot 2 \cdot 1} = \frac{f^4(0)}{4!}$

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$$f(x) \approx f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^4(0)}{4!}x^4 \dots$$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^n(0)}{n!}x^n$$

Maclaurin Series

find the Mac series for $f(x) = e^x$

$f(x) = e^x \quad f(0) = 1$

$f'(x) = e^x \quad f'(0) = 1$

$f''(x) = e^x \quad f''(0) = 1$

$\vdots \quad \vdots \quad \vdots$

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} \dots$$

let $x = 1$

$$e = 1 + 1 + \frac{1}{2} + \frac{1}{3!} + \frac{1}{4!} \dots$$

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$f(x) = \sin x \quad f(0) = 0$

$f'(x) = \cos x \quad f'(0) = 1$

$f''(x) = -\sin x \quad f''(0) = 0$

$f^3(x) = -\cos x \quad f^3(0) = -1$

$f^4(x) = \sin x \quad f^4(0) = 0$

$\vdots$

1

0

-1

$\vdots$

$$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 \dots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \dots$$

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